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The chart showing the process of definition reviews the individual steps in this discussion. It should be emphasized that in developing the notion of probability it is necessary to recognize the combined mathematical and empirical nature of the problem. It is conceded that mathematical points, lines, and planes are abstractions. They have neither been observed in the physical world, nor can they be visualized, and thus far, they do not exist outside the paradise of the student of pure mathematics. The relation between the abstract geometrical elements and the corresponding graphical or physical elementary objects is that they are at most mutual approximations. Successful geometry has been developed in spite of that or perhaps on account of that. Abstract probabilities might be derived by eliminating all but the merely mathematical qualities of probability. But in the field of probability the gulf between abstraction and reality is less transparent, and its bridging by proper methods of definition is of decided importance.

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RECENT WORK IN MATHEMATICAL LOGIC.

A delightful simplification of the primitive propositions required in Russell's logic (see *Principia Mathematica*, Vol. I, 1910) is made by J. G. P. Nicod in a graceful piece of work (*Proc. Camb. Phil. Soc.*, 1916, XIX, 32-41). It will be remembered that four functions of propositions are used in Russell's logic—not- p , p or q , p and q , p implies q : of these, two are taken as indefinables. Nicod makes use of Sheffer's idea (*Trans. Amer. Math. Soc.*, XIV, 481-488) using " p stroke q " to mean "not both p and q " and defines the four functions ordinarily used in terms of this one indefinable. The stroke may be called the sign of *incompatibility*. By means of *three* primitive propositions the propositions required for mathematical logic are developed. The primitive propositions are as follows:

1. If p and q are elementary propositions, so is p stroke q .
2. If p and p stroke (r stroke q) are true, then q is true.
3. P stroke (π stroke Q), where P stands for p stroke (q stroke r), Q for [s stroke q] stroke [$(p$ stroke $s)$ stroke (p stroke s)] and π for t stroke (t stroke t).

The generalized form of the principle of inference (the second

of these propositions) is interesting and deserves notice. The third of these primitive propositions is of course exceedingly complicated and does not commend itself as true so readily as the primitive propositions of *Principia Mathematica*, but it is very interesting to have found it and to have shown how the more familiar propositions can be deduced. In the same issue of the *Proc. Camb. Phil. Soc.* C. E. van Horn uses the same indefinable calling it " p deltas q ." Then from a certain group of three primitive propositions he claims to be able to deduce the propositions required for logic. These propositions consist of the principle of inference as given in *Principia Mathematica*, a proposition analogous to prop. 1.71 in *Principia Mathematica*, by means of which it is possible to deduce that p deltas q is an elementary proposition, when p and q are elementary propositions; and a third proposition stating that if p and q are of the same truth value, p and p deltas q are of opposite truth values, and that if p and q are of opposite truth values, p deltas q is true. However, in the development of this system, it is clear that some axiom is required to connect "deltas" with "of the same truth value." It is not difficult to find vicious circles in some of the demonstrations (cf. the criticism of Nicod at the end of his paper, *ibid.*, p. 40).

C. I. Lewis (*Journal of Phil., Psy., and Scientific Methods*, 1917, XIV, 350-355) gives a further statement of his views on the nature of material implication (cf. *Mind*, 1912, number 84; 1914, number 90) and continues his criticism of Russell's use of the notion.

A translation of part of Frege's *Grundgesetze* is given in *The Monist* (1917, XXVII, 114-127). A. E. Heath (*ibid.*, pp. 1-56) contributes an interesting account of Grassmann's work. D. M. Wrinch (*ibid.*, pp. 83-104) gives a sketch of Bernard Bolzano's life and his pioneer work in mathematical logic. Philip E. B. Jourdain (*ibid.*, pp. 142-151) discusses existence and distinguishes the "entity" of a number from its "existence."

N. Wiener (*Trans. Amer. Math. Soc.*, 1917, XVIII, 65-72) attacks a very general problem in Boolean algebras, finding the necessary and sufficient conditions that a relation between any number of elements will remain invariant with reference to all transformations of the algebra into itself which may be expressed in the symbolism and which leave it a Boolean algebra.

An extract from Couturat's unpublished *Manuel de Logistique* is given in the *Rev. de Métaphys.* (1917, XXIV, 15-58). The

nature of propositional functions, formal and material implication, and real and apparent variables is examined. The article concludes with a sketch of the more elementary parts of the calculus of classes and the definition of 0 and 1. Part of an unfinished treatise written by Couturat sometime before 1902 is given in the same periodical (*ibid.*, pp. 291-313), which is interesting as an exposition of the *frequency* view of probability originally put forward by Venn and the mathematical writers, as opposed to the *sufficiency* view held by Bosanquet, Lotze, and Sigwart. *Probable* is said to be significantly predicable only of indeterminate judgments (by which is meant, presumably, propositional functions) and not of events or determinate judgments. The probability of a judgment is then defined to be the ratio of the number of cases when it is true to the number of cases when it is true or false. The article ends with a short account of the elementary part of the calculus of probabilities. Tenney L. Davis (*Journal of Phil., Psy., and Scientific Methods*, 1917, XIV, 421-440) gives an interesting study of the theory of probabilities, discussing some of the philosophical problems as well as the calculus.

F. Enriques (*Rev. de Métaphys.*, 1917, XXIV, 149-164), in an article on the mathematical infinite draws a distinction between a *potential* infinite and an *actual* infinite. As an illustration he gives the set

$$\cdot 3, \cdot 33, \cdot 333, \dots$$

Now, this ordered set can be given in two ways. (1) The general term consists of so many threes and the terms are ordered by the relation "less than." (2) Each term is obtained from the term before, the first term being $\cdot 3$. In the first case we are said to have an *actual* and in the second case a *potential* infinite. The real antithesis, however, seems to be that in the second case we have the terms given as the field of a relation which is one-one, relating only consecutive terms, whereas in the first case the terms are given as the field of a relation which is transitive and symmetrical, and thus relates any early term to any later one. In his brilliant work on the *ancestral* relation (see *Begriffsschrift*, 1879, Part III, pp. 55-87, *Grundgesetze der Arithmetik*, Vol. I, 1893, §§ 45-46), Frege has shown how to manufacture a relation having the formal characteristics of a series from a one-one-relation: "less than" is the ancestral relation obtained from the relation between terms in (2). There seems to be no fundamental logical distinction with respect

to the nature of the infinite as Enriques suggests. The question of the validity of the assumption of the existence of classes is discussed, but no mention is made of the work of Whitehead and Russell (cf. *Principia Mathematica*, Vol. I) which suggests a method of logical construction by means of which this assumption can be avoided.

A. Padoa (*Rev. de Métaphys.*, 1917, XXIV, 315-325) discusses the general problem of changing the primitive ideas in a deductive system. In an interesting article on the function of symbolism in mathematical logic (*Scientia*, 1917, XXI, 1-12) Philip E. B. Jourdain answers certain charges brought by Rignano against mathematical logic (*ibid.*, 1916, XX). He points out that, though the aim of symbolism in mathematics and logic has been until recently to make the process of reasoning mechanical by pointing out algebraic *analogies*, the aim in modern logic has been to increase the subtlety of our thought by emphasizing *differences*. An account of the work of Peano, Frege, and Russell is given.

In an article, entitled "The Organisation of Thought" in a book of the same title (London, 1917), Prof. A. N. Whitehead investigates the relation between science and logic, and describes some of the features of modern logic.

Prof. L. P. Saunders (*Mind*, 1917, number 101) in a criticism of Mr. Russell's *Lowell Lectures* discusses the nature of logical constructions. The subject is interesting to the mathematical logician, in that, on the validity of this method rests the validity of all but the most elementary parts of mathematical logic, for classes, relations, and numbers are all logical constructions. D. M. Wrinch (*ibid.*, 1917, number 104) attempts to answer some of the points brought up by Professor Saunders in his attack.

Raphael Demos (*Mind*, 1917, number 102) contributes an interesting article on particular negative propositions. He assumes that there are no negative facts and deduces that negative propositions must have reference to the world of positive facts. Particular negative propositions are said to be *ambiguous descriptions of positive propositions* and they are treated as Russell treats descriptive phrases (see *Principia Mathematica*, Vol. I). It seems, however, to the reviewer a little unwise to base a theory on such a disputable point as the non-existence of negative facts.

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